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13. ABSTRACT (Maximum 200 words)

This is the Final Report of a three year (plus no-cost extension) research grant to study GPS. Techniques used were developed on earlier AFOSR/NM grants, and include inverse problems, wavelets, information theory, and geodesy. This project developed new signal processing methods, either: (i) to improve the accuracy of the GPS, (ii) to generalize GPS to other planets and moons in our solar system or even beyond, or (iii) to detect and defeat tampering with the security of the GPS. Jamming, spoofing and unwanted access can come from human sources or atmospheric conditions. The multipath reduction paper discusses topics (i) and (iii), while the Feynman functional and geodesy papers discuss all the topics above, (i)-(iii). The gravity detection work in progress gives markers to detect tampering with GPS, as well as being very important to fundamental physics and geophysics.

14. SUBJECT TERMS

Inverse problems, wavelets, GPS, Information theory, Geodesy

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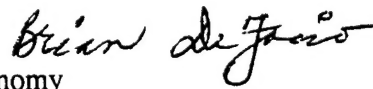
FINAL REPORT SF 298
AFOSR/NM Grant F 49620-~~99-0-1078~~ 99-1-0178

June 20, 2002

Inverse Problem Research to Improve the
Global Positioning System

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1.   OBJECTIVES

The goals of this project were to perform research in fundamental mathematics and physics, which will improve the operation of the GPS by understanding it better. Some of the methods being used include Feynman "integral" functionals, wavelets, information theory, and photonic band-gap structures- PBGS

All of our calculations are evaluated in the full GPS system, by using the MATLAB toolbox Constell Constellation that simulates the entire GPS for aircraft navigation. During the second half and during the no cost extension of this grant we have focussed our work on the study of developing results which address security aspects of the GPS. The broadband, low energy design, which makes GPS so accurate and so effective, also makes it vulnerable to jamming, spoofing and deniable access. The 20.46 MHz bandwidth of the military P(Y) code signal L1 can use relativistic effects and known random earth markers to detect jamming and spoofing signals. Then filters special antennas or a different idea are needed to remove these distortions.

Another product of these studies is the improvement of Ph.D. and undergraduate education, by including research students in this project at both levels. It is a false dichotomy that science and technology can be neatly divided into "pure" and "applied", and it is an even bigger lie that education and research are separate endeavors

2.   STATUS OF THE EFFORT

So far, four publications have appeared, others are in preparation, invited talks were presented in Seoul, Korea (July 1999) and at the Washington, D.C. national meeting of the American Mathematical Society (January 2000) plus a colloquium and a seminar were presented to the mathematics department at the University of Nebraska-Lincoln (March 2000). In 2001-2 several invitations were declined, e.g. Universities of Miami and Nebraska because the PI was too busy and an error in the Missouri grant office shifted the completion date of this project from early April 2002 to November 30, 2001. This latter error led to having to decline 3 workshop invitations, which would have been a

very good ending to this project, even with a generous 6 month no cost extension. The missed meeting includes the AFOSR/NM Program Review on the," Cutting edge of signal processing," in Burlington, VT. From June 6-8, 2002; the International Association of Relativistic Dynamics, IARD, Biennial meeting in Washington DC June 24-26, 2002 and the American Mathematical Society Summer Research Conference on "Waves in Periodic and Random Media" held from June 22-28, 2002 at Mount Holyoke in South Hadley, MA. The overlap in the last two would have been met by the PI attending the entire IARD conference and Ph.D. student Arjuna Flenner would have presented our talk on the convex huts of periodic materials, which by design can filter out the multipath signals from entering GPS ground stations. Had this plan been used, DeFacio would have then gone to the AMS meeting for the last two days.

There are three other invitations that I intend to accept and to present some of our GPS results that we have obtained on this grant including:
1. "Inverse Problem Methodology in Complex Statistical Models," Statistical and Applied Mathematical Sciences institute, SAMS, a new NSF institute located at Research Triangle. NC September 21-24, 2002;
2. "Conference on Mathematical Modeling of Wave Phenomena," Vaxjo University, Sweden November 3-8, 2002;
3. "The Feynman Integral Along with Related Topics and Applications," Mathematical Sciences Research institute, MSRI, University of California, Berkley, December 9-12, 2002.

The areas of GPS that we are actively investigating include the satellite orbital motions, the sources of noise and perturbations and control methods based on improved understanding of the Satellite trajectories. Wavelets and their frames are being used to analyze the GPS residuals to study deviations from normal distributions. After all, many modern physical processes have not existed for enough relaxation times for the Central Limit Theorem to provide the long time Gaussian probability distribution function, (the normal distribution) to develop. The n-dimensional wavelet structure with frequency scales and space-time variables gives additional scale information on the residuals of the signals so they can be modeled out. We are also using information theory [1] in these studies to model the residuals more precisely and thereby to further reduce them. This modeling includes the effects of satellite motions, with gravitational perturbations and atmospheric perturbations on the signals [3]. The information theories involve generalized entropy functionals beyond Shannon informations, such as Kullbach-Liebler and Karhunen-Loeve informations.

We were not successful in obtaining a transition on this research grant. A number of groups were contacted including the Sensors group at the Wright Patterson AFB, JPL groups on geophysics and clocks in space systems at the California Institute of Technology, the Boeing Aircraft navigation and Allied Signals groups in St. Louis and a Tribble Corporation in New Zealand. Most of these had some interest but none of these attempts worked out. Greg Jones patent application remains active so it can lead to some future transition. However, I know of no activity on it at present.

3.   ACCOMPLISHMENTS

We have made some progress on the GPS research, based on previous work on other AFOSR research projects. Information theory is used throughout our work [1]. Both some Feynman (integral) functional and some photonic band-gap structure studies have already proven useful and have yielded two publications. The Feynman functional gives a rigorous formulation of some GPS gravitational and geodetic problems, in a way that many calculations can be performed. This requires a better choice of local coordinates for the numerous rotational motions involved, which may be the reason that it doesn't seem to be known. Much of this work is complete and parts of it are in manuscripts in preparations [2-6]. The PBGS provide a new class of filters that can removal many dB of unwanted multipath signals at the ground receivers [2]. We have another idea for removing the multipath signals started as well.

My former undergraduate mentor student Greg Jones is truly exceptional. He has discovered a new elementary proof (using the mean value theorem) of the convergence of the iterative series for the geodetic parameters for the deformed ellipsoid of the earth [4]. He has now improved this work by finding a new second order convergent method and has tested an algorithm. which is faster and more general than existing ones. His work includes both proofs and calculation. Greg has obtained a patent disclosure through Missouri University. Since his patent application has been registered he has submitted his paper and it is accepted. A draft was enclosed with last year's report. The proofs and calculations are valid at large orbit eccentricity, i.e. large deviations from a circular shape. He also found a clever way to smooth the iteration terms for large deformations. These new cases are not yet needed for planet to earth geodesy, but will be essential for future navigation and communication technologies which operate beyond the earth-moon system. He will publish this work alone in the Journal of Geodesy and later we will submit a more general proof together. An excellent Ph.D. candidate. Arjuna Flenner and I are working on wavelet analysis. noise analysis and relativistic effects of gravity. We have proposed a PBGS filter to remove many of the signals from the multipaths [2], and

now are using the ideas with geometry for security filters. We also have a different idea for removing multipath signals.

The noise analysis is interesting because engineers and scientists often assume a Gaussian distribution of probabilities without sufficient justification. In several more mature problems such as ultrasound, quantum optics, communications, and pulsar timing, a very different situation is found. Parts of the noise spectrum are clearly different from a Gaussian distribution; at different frequencies this is called "colored" noise with the Rayleigh distributions as an example. For these processes, low frequency divergences occur if sufficient care is not taken (the probability is zero for any zero frequency event which occurs in a finite time.) Wavelet scales were useful for a related problem in ultrasound, which we worked on in reference [1]. In these cases the time domain can be used, and only the non-stationary (time) parts are physically meaningful, since the frequency domain is limited to stationary processes, but the wavelet scale variable is valuable for these non-stationary processes. Thus, time is fundamental in GPS, geodesy and space physics. The atomic clocks aboard the GPS Satellites are highly accurate for short times of order of a few (earth) hours but are subject to drift at longer times. Several binary pulsars are known to be equally highly accurate for time periods well over 1000 hours. Some of our work addresses monitoring and correcting these drifts. Improvements in this area are germane to the large scale tests of general relativity, but are equally important to the space navigation and communication interests of the Air Force.

One of these studies [5] is a study of LISA signals using wavelets and information theory. The LISA, Laser Interferometer Space Antenna, system consists of six satellites orbiting the earth in low eccentricity (nearly circular) paths. The earth satellite plane is perpendicular to the instantaneous tangential velocity of the earth in its motion about the sun. This system has been under design since 1994 and is planned for full operation in 2008. It is jointly supported by the European Common Market nations and NASA. The principal scientific objectives of the LISA project are to study a low frequency band, 10E-4 Hz – 10E-1 Hz, of gravitational waves. These effects cannot be measured by any of the ground-based detectors that exist today or even any that are in the design stages.

The new physics that LISA is designed to study includes:
- Galactic binary systems. which contain black holes, neutron stars and white dwarf pairs. These are the most probable ancestors of the important Type I Supernova. These objects are key elements of the possible recent large distance acceleration in the Hubble velocity at very large distances.

• Mergers of the massive black holes that are believed to be present in many galactic centers.

• Interactions between black holes with massive black holes. These studies are good candidates to provide precise data on strong nonlinear gravitational field effects.

• Discrimination between different proposed mechanisms for massive black holes.

There is an American project of gravitational wave managed by MIT and Caltech called Laser Interferometric Gravitational Wave Observatory, abbreviated LIGO. This project consists of two L-shaped elastic bars 7 km long. The frequency response of LIGO is from 50-10,000 Hz and careful studies of sharp impulsive signals, monochromatic signals, chirp signals, and localized pulses, many of which resemble P. Levi's distributions, are being investigated. We were able to get several Mega Bites of data in May 2001 through Professor A. Lazzarini of Caltech, who is head of the Data Analysis group of LIGO. The LIGO group loaned us their instrument transfer function in June after we sent them some of our output. We are using the transfer function, the LIGO data and the astrophysical theory present by Dr. Cutler of the German group to develop signal processing algorithms for LISA. These algorithms are heavily based on Wavelet information theory. The publications of these and related studies may be slow to appear since many people and thirty years of scientific work by some of them are involved.

There is a joint French-Italian project called Virgo, which has different experimental design parameters that will cover a different frequency window with some overlap with the low frequencies of LIGO, but none with LISA. Using reasonable values from observational Astronomy, LIGO is far more sensitive to neutron stars with mass 1.4 times the mass of the sun, than to black holes with masses ten times the mass of the Sun. It also has a better sensitivity to stochastic pulse, impulsive signals and monochromatic gravity waves as well.

Our colleague and friend professor Sergei Kopeikan is friends and collaborates with Professor A. Giazotto of the University of Pisa, the co-chair of the VIRGO project. Sergei has agreed to help us get as much of this data as we need.

This information is physically important, but it is also a key part of Air Force navigation and communication. Our analysis is valid for both LISA satellites and for ground-based detectors that allows us to look at these ripples on the gravitational curvature in more

than one way. In addition to improving GPS accuracy, we will look for natural markers whose distortion marks spoofing or jamming in GPS signals so the ground station can remove that satellite until the marker becomes normal again

The study in ref. [6] was part of the original proposal for this project as extended by the work of Jones [4] in our group. We study the gravitational field of the earth and then consider very different planets with larger eccentricities. This work is an example of an application of the GPS to future problems. This information will be required for the planning of solar system navigation and communications by the Air Force including both unmanned and manned missions.

We are now extending our Lund University Press paper [2] to various new zonotopes, i.e. convex bodies with $2^n*n!$ faces. These include the Buckey ball suggested there plus many new geometric shapes. For example, geometric shape of different complexity can be used in different situations depending on the safety of the location where it is being used. Smaller simpler frequency filters could be constructed so that they could be destroyed by remote control to avoid their capture.

4. PUBLICATIONS

1. A.J. Van Nevel, B. DeFacio and S.P. Neal, "Information-theoretic wavelet noise removal for inverse elastic wave scattering theory," Phys. Rev. E 59, 3682-3693 (1999).

————————————————————Year One Only————————————————————

2. S. Chakraborty, A. Flenner and B. DeFacio, "Two-dimensional photonic band-gap structures for the reduction of multipath corrections to the Global Positioning System," Festschift to Professor Staffan Ström, (Lund University Press, Sweden, 1999), pp 45-62.

3. B. DeFacio, "A Feynman functional for the Global Positioning System," J. Korean Math. Soc. 38, 321-336 (2001)

4. G.C. Jones, "New solutions for the geodetic coordinate transformation," (accepted, in press, Journal of Geodesy and patent application submitted).

5. A. Flenner and B. DeFacio, "Wavelet analysis of gravitational signals for detections by LISA," (in preparation).

6. A. Flenner and B. DeFacio, "Wavelet detection for gravitational wave studies in the LIGO data" (in preparation.)

7. A. Flenner and B. DeFacio, "Doppler Geodesy from Large Eccentricity Planets," (in preparation).

5.   INTERACTIONS

1. Participant in the AFOSR Surveillance and Communication, Mathematical and Space Sciences, Contractors' Meeting and Spring Technical Review. Presented a research report entitled, "Information, Inverse Problems and Noise Alleviation." April 14-16, 1999 at Minnowbrook Conference Center, New York.

_____Year One Only_____

2. Visited the AFIT Department of Mathematics with Professor Mark Oxley, Host. Presented a talk on the Feynman Functionals for the GPS on May 26, 1999. Jim Leonard, Peter Howe and a couple of more people from the Sensors Directorate attended and we talked after my seminar.

3. Invited Paper at the Conference on Feynman Integrals and Related Topics at Younsei University, Seoul, Korea 10-17 July 1999. Refereed article on this topic published by the J. Korean Mathematical Society, **38,** 321-336 (2001).

4. Invited refereed paper in the Retirement Festschrift for Professor Staffan Strom, Professor of Electromagnetics, of the Royal Insititute of Technology, KTH, Stockholm, Sweden. Edited by Professor Dr. Gerhard Kristensson and Dr. Anders Karlsson, Department of Theoretical Electromagnetics, Lund University, Lund, Sweden (appeared December 1999), pp. 45-62.

5. Invited paper presented at the Washington, DC, Meeting #950 in the special session on Feynman Integral and Applications, Co-chairs G.W. Johnson and Michel Lapidus. Talk title: Application of a Feynman Functional to the Global Positioning System," Brian DeFacio and Arjuna Flenner, January 20-23, 2000.

6. Invited Mathematics Department Colloquium, University of Nebraska-Lincoln, NE. "Some Mathematics of the GPS," March 29, 2000.

7. Invited Mathematical Analysis Seminar, University of Nebraska-Lincoln, NE. "On the Motion of a GPS Satellite." March 30, 2000.

6.   ADDITIONAL MATCHING FUNDS

Several opportunities arose during both years to obtain additional matching funds for this work beyond those in the original proposal including:

Additional funds obtained 2001-2002	$18,400.00
Additional funds obtained 2000	$1,100.00
Additional funds obtained 1999	$8,275.00
Total additional funds	$27,775.00

7.   HONORS

Fellow of the American Physical Society since 1994.

Listed in Marquis Who's Who in Engineering and Science since 1998.

Listed in Marquis Who's Who in the World since 2000.

Listed in Marquis Who's Who in America since 2001.

Gregory Chapman Jones, undergraduate student working on this grant, received a $7,500.00 Goldwater Fellowship (One of 120, nationally). Greg now has an NSF Fellowship at Harvard University.

8.   TRANSITIONS

There were no transitions during this grant. Several possibilities were attempted, but none of them were successful.